Supplementary Material to "Towards a non-invasive diagnosis of portal hypertension based on an Eulerian CFD model with diffuse boundary conditions"

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1 Numerical Implementation

At the beginning of simulation, both the velocity and pressure fields are initialized to be zero. The transient term $\rho\frac{\partial \mathbf{v}}{\partial t}$ will be temporarily integrated into the momentum equation to update the velocity field. Therefore, the following governing equations will be solved in our numerical implementation

$$\begin{aligned} \rho\frac{\partial \mathbf{v}}{\partial t} &= -\nabla p + \mu\Delta\mathbf{v}, \\ \nabla\cdot\mathbf{v} &= 0. \end{aligned} \tag{1}$$

Note when both the velocity and pressure fields converge to their final solutions, the transient term $\rho\frac{\partial \mathbf{v}}{\partial t}$ will finally converge to $\mathbf{0}$. To solve above equations, the semi-implicit method for pressure-linked equations (SIMPLE) algorithm is applied and its full procedure is as follows [1]

1) Initialize both pressure field p^* and the velocity field $\mathbf{v}^*$ to be zero;
2) Solve the following momentum equation to obtain $\mathbf{v}'$

$$\rho\frac{\mathbf{v}' - \mathbf{v}^*}{\delta t} = -\nabla p^* + \mu\nabla^2\mathbf{v}', \tag{2}$$

   where δt is the time step size.
3) Solve the following pressure Poisson equation (PPE) to obtain p'.

$$\nabla\cdot\frac{\delta t}{\rho}\nabla p' = \nabla\cdot\mathbf{v}', \tag{3}$$

4) Add p' to p^*;
5) Return to step 2 and repeat the whole procedure until the converged solution is obtained
6) Set $\mathbf{v}^* = \mathbf{v}'$, return to step 2 and repeat until the transient term vanishes.

1.1 Discretization of the momentum equation

For simplicity, we only give the discretized formulations for a one-dimensional problem. Consider a cell that is located inside the vessel but far away from the boundary, its momentum equation can be discretized as

$$\frac{u_i^{'} - u_i^*}{\delta t} = \frac{\mu}{\rho} \frac{u_{i+1}^{'} - 2u_i^{'} + u_{i-1}^{'}}{d^2} - \frac{1}{\rho} \frac{p_i^* - p_{i-1}^*}{d}. \tag{4}$$

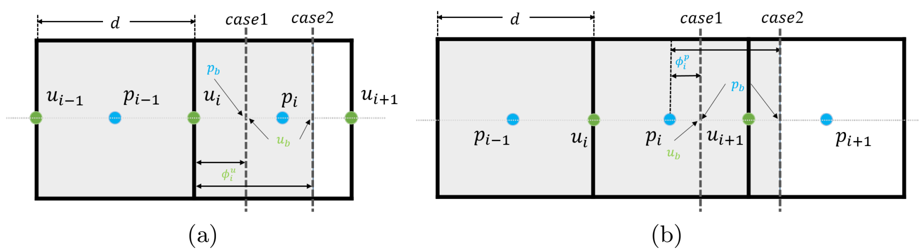

Fig. 1. Discretization. (a) discretization of the momentum equation; (b) discretization of the pressure Poisson equation.

When the cell is near the boundary, two different cases should be considered. As shown in Fig. 1(a), if the boundary is located between u_i and p_i, the discretized momentum equation is written as

$$\begin{gathered} \frac{u_i' - u_i^*}{\delta t} = \frac{\mu}{\rho} \frac{A_i u_b + (2 - A_i)\, u_{i-1}' - 2u_i'}{d^2} - \frac{D_i}{\rho} \frac{p_b^* - p_{i-1}^*}{d}, \\ A_i = \frac{2d}{d + \phi_i^u}, \quad D_i = \frac{2d}{d + 2\phi_i^u}, \end{gathered} \tag{5}$$

Otherwise, if the boundary is located between p_i and u_{i+1}, the discrete momentum equation is written as

$$\begin{gathered} \frac{u_i' - u_i^*}{\delta t} = \frac{\mu}{\rho} \frac{A_i u_b + (2 - A_i)\, u_{i-1}' - 2u_i'}{d^2} - \frac{1}{\rho} \frac{p_i^* - p_{i-1}^*}{d}, \\ A_i = \frac{2d}{d + \phi_i^u}, \end{gathered} \tag{6}$$

After assembling all equations, the conjugate gradient least squares (CGLS) algorithm in Eigen was applied to solve the linear system of equations.

1.2 Discretization of the pressure Poisson equation

If a cell is located inside and far away from the vessel, the discretized pressure Poisson equation can be written as

$$\frac{\delta t}{\rho} \frac{p_{i+1}' - 2p_i' + p_{i-1}'}{d^2} = \frac{u_{i+1}^* - u_i^*}{d}, \tag{7}$$

When the cell is located near the boundary, its formulation depends on the location of the boundary. If the boundary is located between p_i and u_{i+1}, the discrete pressure Poisson equation is written as

$$\begin{aligned} \frac{\delta t}{\rho} \frac{B_i p_b + (2 - B_i)\, p'_{i-1} - 2p'_i}{d^2} &= C_i \frac{u_b - u'_i}{d}, \\ B_i = \frac{2d}{d + \phi_i^p}, \quad C_i &= \frac{2d}{d + 2\phi_i^p}, \end{aligned} \tag{8}$$

Otherwise, if the boundary is located between p_i and u_{i+1}, the discretized pressure Poisson equation is written as

$$\begin{aligned} \frac{\delta t}{\rho} \frac{B_i p_b + (2 - B_i)\, p'_{i-1} - 2p'_i}{d^2} &= \frac{u'_{i+1} - u'_i}{d}, \\ B_i &= \frac{2d}{d + \phi_i^p}, \end{aligned} \tag{9}$$

The linear system of equations can be solved with the CGLS algorithm as well.

References

1. Liu, S., He, X., Wang, W., Wu, E.: Adapted simple algorithm for incompressible sph fluids with a broad range viscosity. IEEE Transactions on Visualization and Computer Graphics pp. 1–1 (2021). https://doi.org/10.1109/TVCG.2021.3055789