A Direct Variational Pressure Estimation Approach for Velocity Data in 4D Flow MRI

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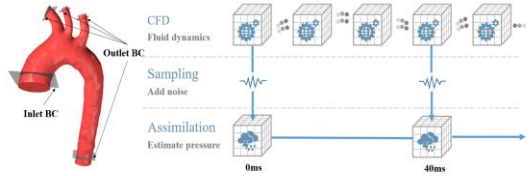
HYPOTHESIS

Blood is modeled as an incompressible inviscid fluid governed by the following Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{\rho} \nabla p, \text{subject to } \nabla \cdot \mathbf{u} = 0$$

$\mathbf{u}$: blood velocity
p: blood pressure
$\rho = 1060 \mathrm{kg}/m^3$: blood density



Overview of the estimation-simulation workflow.
Left: A segmented aortic arch is selected as the vessel wall for blood simulation. Velocity profiles extracted from the in vivo measurements and zero pressure were used as inlet and outlet boundary conditions (BCs), respectively;
Right top: A standard finite volume method (FVM) was used to generate velocity and pressure sequences over one cardiac cycle;
Right middle: Noise was added to the velocity fields to mimic experimental data with uncertainty;
Right bottom: A direct variational method to estimate pressure from two successive velocity fields.

METHODS

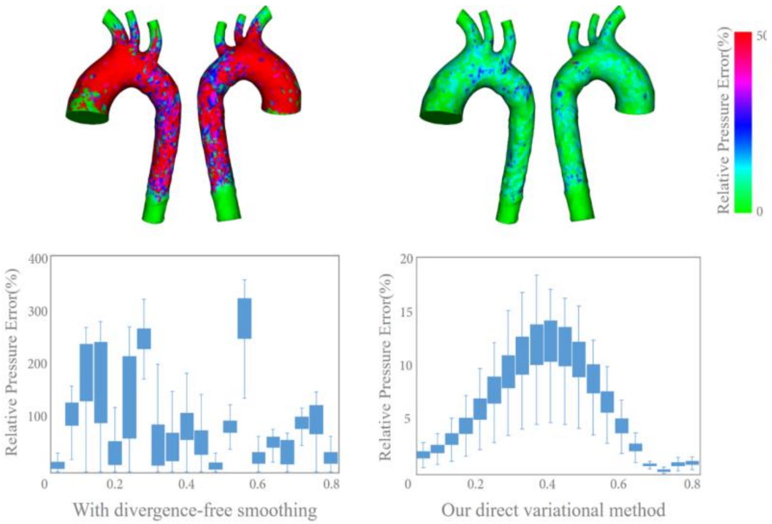
To estimate the pressure field based on two successive velocity fields, a variational energy formulation is proposed as follows

$$\mathrm{E}^{n+1} = \left(\mathbf{u}_s^n - \frac{\delta t}{\rho} \nabla p\right)^T \mathbf{W} \left(\mathbf{u}_s^n - \frac{\delta t}{\rho} \nabla p\right) + \lambda \left(\mathbf{u}_s^n - \frac{\delta t}{\rho} \nabla p - \mathbf{u}_s^{n+1}\right)^T \mathbf{W} \left(\mathbf{u}_s^n - \frac{\delta t}{\rho} \nabla p - \mathbf{u}_s^{n+1}\right)$$

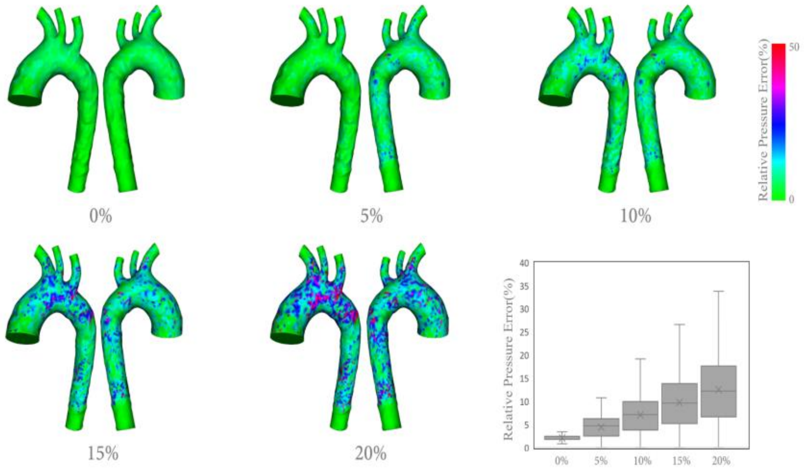
$\mathbf{u}_s^n$: the extracted velocity field at time n
$\mathbf{u}_s^{n+1}$: the extracted velocity field at time n+1
δt: the time interval (which is 40ms in our current setting)
$\mathbf{W}$: a diagonal matrix, each entry representing the fraction of blood occupying each voxel
λ: a positive control parameter to balance the effects of above two terms
E^{n+1}: minimizing the first term respects that pressure p is equivalent to enforcing $\mathbf{u}_s^n$ to be divergence-free, minimizing the second term guarantees that final velocity should not deviate too far from the exacted velocity $\mathbf{u}_s^{n+1}$.

RESULT

Top: A comparison of the relative pressure error distribution at t=0.36s;
Bottom left: Time history of relative pressure error when divergence-free smoothing is taken before pressure estimation;
Bottom right: Time history of relative pressure error with our direct variational method.



With divergence-free smoothing
Our direct variational method



0% 5% 10% 15% 20%

The pressure estimation accuracy with different intensities of white noise were examined at t=0.36s, corresponding to white noise intensities of: 0%(i.e., no noise), 5%, 10%, 15% and 20%.

DISCUSSION

Computational results for pressure show that the accuracy of pressure estimation could be largely affected by noise. Velocity smoothing helps to reduce the divergence error in velocity fields, nevertheless, it does not help improve the accuracy in computing the pressure field according to our experiments. Our direct variational pressure computing method shows improved accuracy in resisting noise during acquisition. Concerning limitations, this study did not asses the accuracy of our method on actual 4D Flow MRI imaging data yet. Further evaluation is required to confirm the accuracy compared to other boundary conditions, numerical solvers as well as pressure measured by intravascular catheterization.

CONCLUSIONS

A direct variational approach applicable to wall-bounded flow has been developed for computing pressure field from velocity data generated from CFD simulation. The results indicate that it has a good anti-noise capability. It is expected to improve the estimation accuracy of pressure required for the reliable diagnosis of cardiovascular diseases with 4D flow MRI.